Cap Rate Calculations

How do investors determine ROI in an unsteady market?

by Eric B. Garfield, MAI, MRICS, and Matthew T. VanEck

capitalization rate is the overall or nonfinanced return on a real estate investment, akin to the return on total assets in accounting terms. A cap rate is calculated as a mathematical relationship between net operating income and an asset's value. Most commonly cap rates are extracted from transactions of buyers and sellers competing in a marketplace; but they are related to the current state of capital markets as well as the future growth outlook. So how can real estate professionals extract cap rates in today's market, where few sales exist?

Generally, cap rates are derived from real property sales via the formula *cap rate* (R_o) = *NOI* ÷ *value*. In first quarter 2008, this cap rate derivation may have sufficed. However, since then, the conclusions would be misstated not only because of changes in time, but also because of the subprime lending crisis' impact and U.S. capital markets' failure. Thus, real estate professionals not only must be able to interpret market data, but they also must understand the capital markets' effect on cap rates — especially in illiquid markets, where sales data is limited.

Credit Crisis and Cap Rates

The relationship between cap rates and their respective capital markets often is overlooked. Leverage, or the effect of borrowed funds on return on investment, is a key component of a cap rate. Leverage generally varies from market to market and is affected by supply and demand as well as interest rates. As a reminder, it is noteworthy that cap rates and discount rates, or internal rates of return, are not mutually exclusive. A discount rate is a measure of investment performance over a holding period that accounts for risk and return on capital. Cap rates not only account for return on capital, but also return of capital. A discount rate can be built up from a cap rate if income and growth both change at a constant rate. The buildup is derived by the formula Y = R +*CR*, where Y = discount (yield) rate, R = cap rate, and CR = constant rate of change.

Thus, if a market-extracted cap rate is 7 percent and the market constant rate of change is 3 percent, the discount rate is 10 percent. This calculation represents an investor's yield expectations on investment, but not return of investment. Return of investment must be calculated separately.

Since the 2008 financial meltdown, the commercial mortgage-backed securities market essentially has stopped functioning, halting most available financing for commercial real estate. Thus, how is the lack of leverage in determining a cap rate accounted for and how do the pre-crash cap rates differ from the post-crash cap rates? A look at appraisal mathematicians L.W. Ellwood's and Charles B. Akerson's analyses provides a quantifiable explanation.

The Anatomy of a Cap Rate

Cap rate quantification began with Ellwood, who is credited with developing financial valuation models at a time when appraisers commonly were using physical residual techniques such as land and buildings. In 1959, Ellwood published "Ellwood Tables for Real Estate Appraising and Financing," which showed that by analyzing market mortgage terms and equity yields for a particular property, an appraiser could identify a suitable cap rate and thus property value. This valuation technique became known as mortgage-equity analysis. Ellwood's method allowed appraisers to incorporate and explain financing's impact on value.

From his research, Ellwood created a formula that "builds up" a property's cap rate on the basis of assumptions concerning mortgage and equity requirements. Using Ellwood's formula, a cap rate results through application of an investor's equity yield requirements, structure of debt, total change in income over the projection period, and change in total property value over the projection period. The resulting cap rate is then divided by NOI to produce a value estimate that explicitly reflects the property's financial considerations. (See "Ellwood's Formula.")

One flaw of Ellwood's formula is its complexity. It not only requires capital markets knowledge, but also algebraic operations. Several years later, Charles Akerson sim-

Eric B. Garfield, MAI, MRICS, is a director with WTAS, a full-service tax compliance and consulting firm. Contact him at (213) 593-2300 or eric.garfield@wtas.com.

Matthew T. VanEck is a manager with the real estate division of FMV Opinions, Inc., in Irvine, Calif. Contact him at (949) 759-4499 or mvaneck@fmv.com.

Ellwood's Formula

R₀

$$= \frac{Y_{E} - M (Y_{E} + P 1/S_{n} - R_{M}) - \Delta_{0} 1/S_{n}}{(1 + \Delta_{1}J) \text{ or } (K)}$$

- R_0 = cap rate that is used to convert income into value
- Y_E = equity discount or yield rate is rate of return on equity capital
 M = loan-to-value ratio is ratio between a mortgage loan and a property's value
- P = percentage of loan paid off in holding period
- $1/S_n$ = sinking fund factor is an element in yield and change formulas that converts the total change in capital value over the projection period into an annual percentage
- R_M = mortgage capitalization rate or mortgage constant reflects the relationship between annual debt service to the principal amount of the mortgage loan
- $\Delta_{_{\rm O}}$ ~ = change in total property value over the projection period
- Δ_{I} = total change in income over the projection period
- *J* = an income stabilization factor used to convert an income stream changing on a curvilinear basis into its level equivalent
- K = an income stabilization factor used to convert an income stream changing at a constant ratio into its stable or level equivalent

Source: The Appraisal of Real Estate, 13th edition

plified Ellwood's formula by altering the calculations to a series of simple arithmetic steps based on a band of investment calculations in his article "Ellwood Without Algebra," in the July 1970 issue of *The Appraisal Journal*. The Akerson formula uses similar components to build up a cap rate; however, it succeeds in simplifying the steps without sacrificing results. (See "Akerson Format.")

Sensitivity to Leverage

In addition to providing a helpful mortgage-equity valuation technique, Akerson's formula also can be used to illustrate the effects of financial leverage or debt on a particular investment. Leverage can be measured by the loan-to-value ratio (M). An LTV change can increase or decrease the equity return (Ye) depending on the specific terms: The higher the risk to the investor, the higher the equity rate an investor will seek to compensate. Leverage is considered positive when the cap rate is greater than the mortgage cap rate or mortgage constant (Rm), while negative leverage occurs when the cap rate is lower than the mortgage cap rate.

Using the Akerson model, the effect of leverage change on equity yield rates can be illustrated. (See "Akerson Format in Action.") Assume that NOI is level at \$100,000 and the subject property can be financed with a 75 percent loan paid monthly at 8 percent annual interest over 25 years. The required market return on

1		A 1		n Ferr		+ Ctor	-	
_	Incre	ase in LTV to		on Fori %, with C				8%
1		М	x	Rm				
2	+	E	х	Ye				
3		М	х	Ρ	Х	1/Sn		
4	=	r						
5	+/-	Dep/(Gain)	х	1/Sn				
6	=	Ro						
1		0.80	x	0.0926			=	0.0741
2	+	0.20	х	0.1509			=	0.0302
3	-	0.80	х	0.1924	х	0.0490	=	-0.0075
4	=	r						0.0967
5	+/-	0.10	х	0.0490			=	-0.0049
6	=							0.0918
	Cap Rate							9.18%
	NOI						\$100,000	
	Value							\$1,088,955
2	Value Rounded							\$1,088,955 \$1,100,000
2	Rounded	Ake Increase in L1		on For om 75%				\$1,100,000
2	Rounded		V fr					\$1,100,000
- 1 2	Rounded	Increase in L1 M E	V fr	om 75% Rm Ye	to 8	30% with		\$1,100,000
1 2 3	Rounded	Increase in LT M E M	V fr	om 75% Rm	to 8			\$1,100,000
- 1 2	Rounded	Increase in L1 M E	X X X	om 75% Rm Ye	to 8	30% with		\$1,100,000
- 1 2 3	Rounded	M E M r	V fr x x x	om 75% Rm Ye	to 8	30% with		\$1,100,000
1 2 3 4	Rounded + - =	Increase in LT M E M	V fr x x x	om 75% Rm Ye P	to 8	30% with		\$1,100,000
1 2 3 4 5	Rounded + - = +/-	M E M r Dep/(Gain)	V fr x x x	om 75% Rm Ye P	to 8	30% with		\$1,100,000
1 2 3 4 5 6 1 2	Rounded + - = +/-	Increase in LT M E M r Dep/(Gain) Ro	V fr x x x	om 75% Rm Ye P 1/Sn 0.0926 0.1400	to 8	30% with	1 Ye at 14%	\$1,100,000
1 2 3 4 5 6 1 2 3	+ = +/- =	Increase in L1 M E M r Dep/(Gain) Ro 0.80 0.20 0.80	V fr x x x x	om 75% Rm Ye P 1/Sn 0.0926	x	30% with	1 Ye at 14%	\$1,100,000 0.0741 0.0280 -0.0080
1 2 3 4 5 6 1 2	Rounded + - = +/- = +/-	Increase in L1 M E M r Dep/(Gain) Ro 0.80 0.20	V fr x x x x x	om 75% Rm Ye P 1/Sn 0.0926 0.1400	x	3 <u>0% wit</u> h 1/Sn	= =	\$1,100,000 0.0741 0.0280
- 1 2 3 4 5 6 1 2 3 4 5	Rounded + + - = +/- = +	Increase in L1 M E M r Dep/(Gain) Ro 0.80 0.20 0.80	V fr x x x x x x	om 75% Rm Ye P 1/Sn 0.0926 0.1400	x	3 <u>0% wit</u> h 1/Sn	= =	\$1,100,000 0.0741 0.0280 -0.0080 0.0941 -0.0052
- 1 2 3 4 5 6 1 2 3 4	Rounded + + - = +/- = + + - = +/- = +/- =	Increase in L1 M E M r Dep/(Gain) Ro 0.80 0.20 0.80 r	V fr x x x x x x	om 75% Rm Ye P 1/Sn 0.0926 0.1400 0.1924	x	3 <u>0% wit</u> h 1/Sn	= = =	\$1,100,000 0.0741 0.0280 -0.0080 0.0941 -0.0052 0.0890
- 1 2 3 4 5 6 1 2 3 4 5	Rounded + - = +/- = + + - = + - + - + -	Increase in L1 M E M r Dep/(Gain) Ro 0.80 0.20 0.80 r	V fr x x x x x x	om 75% Rm Ye P 1/Sn 0.0926 0.1400 0.1924	x	3 <u>0% wit</u> h 1/Sn	= = =	\$1,100,000 0.0741 0.0280 -0.0080 0.0941 -0.0052 0.0890
- 1 2 3 4 5 6 1 2 3 4 5	Rounded + + - = +/- = + + - = +/- = +/- =	Increase in L1 M E M r Dep/(Gain) Ro 0.80 0.20 0.80 r	V fr x x x x x x	om 75% Rm Ye P 1/Sn 0.0926 0.1400 0.1924	x	3 <u>0% wit</u> h 1/Sn	= = =	\$1,100,000 0.0741 0.0280 -0.0080 0.0941 -0.0052 0.0890
2 3 4 5 6 1 2 3 4 5	Rounded + - = +/- = + - = Cap Rate	Increase in L1 M E M r Dep/(Gain) Ro 0.80 0.20 0.80 r	V fr x x x x x x	om 75% Rm Ye P 1/Sn 0.0926 0.1400 0.1924	x	3 <u>0% wit</u> h 1/Sn	= = = =	\$1,100,000 0.0741 0.0280 -0.0080

Akerson Format

Loan ratio (M) x annual constant (R_M)

- + Equity ratio (1-M) x equity yield rate (Y_E)
- Loan ratio (M) x % paid off in projection period (P) x 1/S_n
- = Basic rate (r)
- + Depreciation or gain x $1/S_n$
- = $Overall cap rate (R_0)$

equity for 75 percent financing is 14 percent, and the property is expected to be sold in year 10, at which time the value is expected to have increased (Δ) by 10 percent. Thus, M = .75, E = .25, Rm = .092618 (The present value per payment of \$1 at 8 percent annual interest, amortized monthly over 25 years), Ye = .14 and Δ = .10. The percentage of loan paid off in the holding period (P) can be determined by dividing the amortization

rate of the 8-percent, 25-year full-term loan by the amortization rate of the 8-percent, 10-year holding-period loan. The percentage of loan paid off in the holding period is thus equal to 19.24 percent. The sinking fund factor (the future value per payment of \$1 amortized annually over 10 years at 14 percent equity investment rate) is 0.0517. In applying the Akerson formula, the resulting overall cap rate is .0918 or 9.18 percent. At a level NOI of \$100,000, the value of the subject property is \$1,100,000 rounded. (See chart 5.)

In this example, if the LTV is increased from 75 percent to 80 percent, the equity yield rate will increase as well from 14 percent to 15.09 percent at the same value estimate and at the same cap rate (chart 1). Since there is greater risk when less money is put down, an investor requires a higher equity yield rate for the same return. If the required equity return is unchanged, a higher value will result due to an increase in leverage and a decline in the cap rate (chart 2). Similar relationships exist with changes in the mortgage constant or equity yield rate. Increases in the mortgage constant produce decreases in the equity yield rate. Thus, leverage analysis is important as risk levels directly impact the returns to equity.

Application in 2009

So what does this mean in the current market? Consider this example: Two apartment properties were sold in July 2008 for \$1 million each. The properties

-	8	Apartme	nt Sa	ale Exam	ple -	Original	Sale	
1		М	х	Rm				
2	+	E	х	Ye				
1 2 3 4	-	Μ	х	Ρ	х	1/Sn		
4	=	r						
5	+/-	Dep/(Gain)	х	1/Sn				
6	=	Ro						
1		0.65	х	0.0773			=	0.050
2	+	0.35	х	0.0898			=	0.031
1 2 3 4	-	0.65	х	0.2365	х	0.0659	=	-0.010
4	=	r						0.071
5	+/-	0.10	х	0.0659			=	-0.006
6	=							0.065
	Cap Rate							6.50%
	NOI						\$64,970	1

				Des			
1		M		Rm			
2	+	E		Ye	4.00		
3	1	М	X	Р	x 1/Sn		
4	=	r					
5	+/-	Dep/(Gain)	x	1/Sn			
6	=	Ro					
1		0.50	x	0.0930		=	0.046
2	+	0.50	x	0.0898		=	0.044
1 2 3		0.50	x	0.3323	x 0.0659	=	-0.010
4	=	r					0.080
5	+/-	-0.05	x	0.0659		=	0.003
6	=						0.083
	Cap Rate						8.38%
0						\$64,970	

the composition of the cap rate is the key to understanding and applying mortgage-equity capitalization. Once the anatomy of the capitalization rate is exposed, the rationale of the method becomes apparent." By making sense of cap rate sensitivity, one gains a better understanding of how changes in financial markets correspond to changes in investment perceptions of the future, and more importantly, where the market seems to be headed in times of economic turbulence.

Akerson Format in Action

sold at cap rates of about 6.50 percent. The properties were financed with new loans at 65 percent of value at interest rates of 6.00 percent for 25 years.

How would these transactions differ if they occurred in June 2009? By example, two lenders still active in the market currently quote 55 to 65 percent LTV ratios with interest rates of 6.50 to 7.50 percent (and rising) for these deals. If investors desire the same equity yields, what are the effects on value?

Based on the transaction terms and cap rates at which the apartments sold in 2008, the respective equity yield rate is about 9.00 percent and the mortgage constant is 7.73 percent at 65 percent LTV, 6.00 percent interest for 25 years, and at a 6.50 percent

cap rate, all else remaining constant (chart 3).

In holding the investor's equity yield rate constant in the current credit crisis scenario, an average increase of interest rates by 100 basis points along with a 5 percent LTV ratio decline results in a 68 basis point increase in cap rates to 7.18 percent. The cap rate increase from market conditions results in a June 2009 value of \$900,000 (rounded); a value decline of 10 percent from the pre-credit crisis scenario value of \$1 million in this example (chart 4).

As revealed through dissection of Ellwood's and Akerson's formulas, a cap rate is more than merely the NOI divided by its selling price. As Akerson said in his *Appraisal Journal* article, "Understanding Problem: A property produces an income of \$100.0

Problem: A property produces an income of \$100,000. The property was financed with a 75 percent loan to be paid monthly at 8 percent interest over 25 years. The property is expected to be sold in year 10 at which time the value is expected to have increased by 10 percent. Equity investors expect a 14 percent return on their investment. What is the property's cap rate and estimated value?

The loan ratio (M) is equal to 75 percent of value or 0.75; therefore, the equity ratio (1-M) is equal to 1 - 0.75 or 0.25. In the problem the equity yield rate (Y_E) is specified at 14 percent and gain at 10 percent or 0.10. However, in this case the annual constant (R_M), paid off in projection period (P), depreciation or gain, and sinking fund factor ($1/S_n$) are yet to be calculated.

Annual constant (R_M) can be calculated based on the mortgage interest rate, frequency of amortization, and loan term. Alternatively, it is also the sum of the interest rate and the annual amortization rate (the ratio of the periodic amortization amount to be amortized). Using a financial calculator, the annual amortization rate of the mortgage loan (8.00 percent interest rate loan at 25 years, monthly payments), is equal to 1.26 percent, while the interest rate is 8 percent, resulting in a mortgage constant of 9.26 percent.

The percentage of loan paid off in the holding period (P) can be determined by dividing the amortization rate of the 8 percent, 25-year loan by the amortization rate of the 8-percent, 10-year holding period loan. The percentage of loan paid off in the holding

			on For 75% an					
	M	x	Rm					
+	E	х	Ye					
-	M	х	Ρ	x	1/Sn			(
=	r							1
+/-	Dep/(Gain)	x	1/Sn					
=	Ro							
	0.75	x	0.0926			=	0.0695	9
+	0.25	x	0.1400			-	0.0350	
-	0.75	х		х	0.0517	=	-0.0075	
=	r						0.0970	
+/-	0.10	х	0.052			=	-0.0052	
=	192002					13	0.0918	
Cap Rate NOI						\$100,000	9.18%	:
Value							\$1,088,955	· ·
Rounded							\$1,100,000	:

period is thus equal to 19.24 percent ([9.26 - 8.00]

 \div [14.56 - 8.00] = .1924).

The calculation for depreciation or gain is the estimated percentage change in total property value multiplied by the sinking fund factor used for the equity growth. In this example, the sinking fund factor is calculated on a 14 percent equity yield and a 10-year hold. The result is 0.0517. Multiplied by 10 percent growth, this becomes 0.0052. As this is a gain, it will be deducted.

Therefore, the cap rate results by the following arithmetic steps:

0.75 (M) x 9.26% (R_M) = 0.0695

+ 0.25 (1-M) x 14.00% (Y_E) = 0.0350

0.75 (M) x 19.24% (*P*) x 0.0517 (1/Sn) = 0.0075 0.0970

- 0.0052
- = 0.0918 or 9.18%

The cap rate via Akerson is equal to 9.18 percent and the estimated value is \$1.1 million rounded (\$100,000 NOI \div 9.18 percent = \$1,088,955). This model also can be used for changing income streams when modified by a *J* or *K* factor, as it is with the Ellwood formula. If the same terms were applied to the Ellwood model, the same result would be reached. Akerson format, however, is the standard due to its simplicity. The calculation (rounded) and value are displayed in chart 5.